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Model building by coset space dimensional reduction in eight-dimensions

Toshifumi Jittoh,[1,*] Masafumi Koike,[1,†] Takaaki Nomura,[1,‡] Joe Sato,[1,§] and Yutsuki Toyama[1,¶]

[1]*Department of Physics, Saitama University, Shimo-Okubo, Sakura-ku, Saitama, 338-8570, Japan*

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We investigate gauge-Higgs unification models in eight-dimensional spacetime where extra-dimensional space has the structure of a four-dimensional compact coset space. The combinations of the coset space and the gauge group in the eight-dimensional spacetime of such models are listed. After the dimensional reduction of the coset space, we identified SO(10), SO(10) × U(1) and SO(10) × U(1) × U(1) as the possible gauge groups in the four-dimensional theory that can accomodate the Standard Model and thus is phenomenologically promising.@ Representations for fermions and scalars for these gauge groups are tabulated.

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I. INTRODUCTION

The Standard Model has been eminently successful in describing the interactions of the elementary particles. A crucial role of this model is played by the Higgs scalar, which develops the vacuum expectation value to give the masses to the elementary particles and to trigger the breaking of the gauge symmetry from $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$ down to $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{U}(1)_\mathrm{EM}$. On the other hand, the most fundamental nature of the Higgs scalar such as its mass is not predictable within the Standard Model. Thus, the search for the nature of this particle is essential both for the confirmation of the Standard Model and for the search for new physics.

The gauge-Higgs unification is an attractive approach to account for the origin of the Higgs scalars [1, 2, 3](for recent approaches, see Refs. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]). This approach counts the Higgs scalars as components of the gauge bosons in the spacetime with the dimension higher than four, and attributes their properties to the physical setups such as the gauge symmetry and the compactification scale of the extra-dimensional space. We consider this idea in the scheme of the coset space dimensional reduction, in which the extra-dimensional space is assumed to be a coset space of compact Lie groups, and the gauge transformation is identified as the translation within this space [1, 21, 22, 23, 24, 25, 26]. This identification determines both the gauge symmetry and the particle contents of the four-dimensional theory.

A phenomenologically promising gauge theory in a D-dimensional spacetime, where $D > 4$, should reproduce the Standard Model after the dimensional reduction. Theories in six- and ten-dimensional spacetime has attracted much attention so far in this regard. The chiral structure of the matter content as in the Standard Model is easy to introduce in these cases, more generally when $D = 4n+2$ [27, 28]. Fermions belonging to a vectorlike representation in $(4n+2)$-dimensional gauge theory can end up in a chiral fermion after dimensional reduction by simultaneously applying the Weyl and the Majorana conditions, which are compatible in this dimensionality. This advantage increases the chance for the higher-dimensional model to be a promising candidate. No theories have been found quite promising, however, for the 6, 10, and 14 dimensional spacetimes [22, 26, 29, 30, 31, 32, 33, 34, 35, 36].

We examine the theories in eight-dimensional spacetimes to search further for promising theories. The dimension of the extra-dimensional space $d = D - 4$ is four in this case, and the small d makes the problem tractable. More importantly, the dimension $D = 8$ is below the critical dimension of the string theories, which may thus supply the ultraviolet completions to models in a spacetime of this dimensionality. On the other hand, we need to confine ourselves to the complex representations for the representations of the fermions, unlike the case of $D = 4n + 2$.

We search for the eight-dimensional gauge theory that leads to the Standard Model, the GUTs, or their likes. We exhaustively search for the possible candidates of coset space S/R and the gauge group G of the eight-dimensional theory. The representation for the gauge bosons is then automatically determined. The representation of the fermions

*Electronic address: jittoh@krishna.th.phy.saitama-u.ac.jp
†Electronic address: koike@krishna.th.phy.saitama-u.ac.jp
‡Electronic address: nomura@krishna.th.phy.saitama-u.ac.jp
§Electronic address: joe@phy.saitama-u.ac.jp
¶Electronic address: toyama@krishna.th.phy.saitama-u.ac.jp

is searched up to 1000 dimensional ones, while even larger representations are avoided lest it should generate numerous unwanted fermions after the dimensional reduction. We also tabulate the representation of the scalars and fermions under the gauge group of the four-dimensional theory.

This paper is organized as follows. In Section II, we briefly recapitulate the scheme of the coset space dimensional reduction (CSDR) in eight dimensions. In Section III, we search for the candidate of models in eight dimensions which lead to phenomenologically promising theories in four-dimensions after the dimensional reduction. Section IV is devoted to summary and discussions.

II. CSDR SCHEME IN EIGHT DIMENSIONS

In this section, we briefly recapitulate the scheme of the coset space dimensional reduction in eight dimensions [22].

We begin with a gauge theory defined on an eight-dimensional spacetime M^8 with a simple gauge group G. Here M^8 is a direct product of a four-dimensional spacetime M^4 and a compact coset space S/R, where S is a compact Lie group and R is a Lie subgroup of S. The dimension of the coset space S/R is thus $4 \equiv 8-4$, implying $\dim S - \dim R = 4$. This structure of extra-dimensional space requires the group R be embedded into the group SO(4), which is a subgroup of the Lorentz group SO(1, 7). Let us denote the coordinates of M^8 by $X^M = (x^\mu, y^\alpha)$, where x^μ and y^α are coordinates of M^4 and S/R, respectively. The spacetime index M runs over $\mu \in \{0,1,2,3\}$ and $\alpha \in \{4,5,6,7\}$. In this theory, we introduce a gauge field $A_M(x,y) = (A_\mu(x,y), A_\alpha(x,y))$, which belongs to the adjoint representation of the gauge group G, and fermions $\psi(x,y)$, which lies in a representation F of G.

The extra-dimensional space S/R admits S as an isometric transformation group. We impose on $A_M(X)$ and $\psi(X)$ the following symmetry under this transformation in order to carry out the dimensional reduction [21, 37, 38, 39, 40, 41]. Consider a coordinate transformation which acts trivially on x and gives rise to a S-transformation on y as $(x,y) \to (x, sy)$, where $s \in S$. We require that the transformation of $A_M(X)$ and $\psi(X)$ under this coordinate transformation be compensated by a gauge transformation. This symmetry makes the eight-dimensional Lagrangian invariant under the S-transformation and therefore independent of the coordinate y of S/R. The dimensional reduction is then carried out by integrating the eight-dimensional Lagrangian over the coordinate y to obtain the four-dimensional one. The four-dimensional theory consists of the gauge fields A_μ, fermions ψ, and in addition the scalar fields originated from A_α. The gauge group reduces to a subgroup H of the original gauge group G.

The gauge symmetry and particle contents of the four-dimensional theory are substantially constrained by the CSDR scheme. We provide below the prescriptions to identify the four-dimensional gauge group H and its representations for the particle contents.

First, the gauge group of the four-dimensional theory H is easily identified as

$$H = C_G(R), \tag{1}$$

where $C_G(R)$ denotes the centralizer of R in G [21]. Thus the four dimensional gauge group H is determined by the embedding of R into G. These conditions imply

$$G \supset H \times R, \tag{2}$$

up to U(1) factors.

Second, the representations of H for the scalar fields are specified by the following prescription. Let us decompose the adjoint representation of S according to the embedding $S \supset R$ as,

$$\text{adj } S = \text{adj } R + \sum_s r_s. \tag{3}$$

We then decompose the adjoint representation of G according to the embeddings $G \supset H \times R$;

$$\text{adj } G = (\text{adj } H, \mathbf{1}) + (\mathbf{1}, \text{adj } R) + \sum_g (h_g, r_g), \tag{4}$$

where r_gs and h_gs denote representations of R and H, respectively. The representation of the scalar fields are h_gs whose corresponding r_gs in the decomposition Eq. (4) are also contained in the set $\{r_s\}$ in Eq. (3).

Third, the representation of H for the fermion fields is determined as follows [42]. The SO(1, 7) Weyl spinor $\mathbf{8}$ is decomposed under its subgroup $(\mathrm{SU(2)_L} \times \mathrm{SU(2)_R})(\simeq \mathrm{SO}(1,3)) \times (\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2)(\simeq \mathrm{SO}(4))$ as

$$\mathbf{8} = (\mathbf{2}_L, \mathbf{1}, \mathbf{2}_1, \mathbf{1}) + (\mathbf{1}, \mathbf{2}_R, \mathbf{1}, \mathbf{2}_2), \tag{5}$$

TABLE I: A complete list of four-dimensional coset spaces S/R with rank$S$ = rank$R$. We also list the decompositions of the vector representation $\mathbf{4}$ and the spinor representation $(\mathbf{2}_1, \mathbf{1}) + (\mathbf{1}, \mathbf{2}_2)$ of SO(4) $\simeq$ SU(2)$_1 \times$ SU(2)$_2$ under the Rs. The representations of r_s in Eq. (3) and σ_{1i} and σ_{2i} in Eq. (6) are listed in the columns of "Branches of $\mathbf{4}$" and "Branches of $\mathbf{2}$", respectively.

S/R		Branches of $\mathbf{4}$	Branches of $\mathbf{2}$
(i)	Sp(4)/[SU(2) × SU(2)]	$(\mathbf{2}, \mathbf{2})$	$(\mathbf{2}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{2})$
(ii)	SU(3)/[SU(2) × U(1)]	$\mathbf{2}(\pm 1)$	$\mathbf{2}(0)$ and $\mathbf{1}(\pm 1)$
(iii)	$(\mathrm{SU}(2)/\mathrm{U}(1))^2$	$(\pm\mathbf{1}, \pm\mathbf{1})$	$(\pm\mathbf{1}, 0)$ and $(0, \pm\mathbf{1})$

where $(\mathbf{2}_L, \mathbf{1})$ and $(\mathbf{1}, \mathbf{2}_R)$ representations of $\mathrm{SU(2)_L \times SU(2)_R}$ correspond to left- and right-handed spinors, respectively. The group R is embedded into the Lorentz (SO(1, 7)) subgroup SO(4) in such a way that the vector representation $\mathbf{4}$ of SO(4) is decomposed as $\mathbf{4} = \sum_s r_s$, where r_ss are the representations obtained in the decomposition Eq. (3). This embedding specifies a decomposition of the spinor representations $(\mathbf{2}_1, \mathbf{1})((\mathbf{1}, \mathbf{2}_2))$ of SU(2)$_1 \times$ SU(2)$_2 \supset R$ as

$$(\mathbf{2}_1, \mathbf{1}) = \sum_i (\sigma_{1i}) \quad \left((\mathbf{1}, \mathbf{2}_2) = \sum_i (\sigma_{2i}) \right). \tag{6}$$

We now decompose representation F of the gauge group G for the fermions in eight-dimensional spacetime. Decomposition of F is

$$F = \sum_f (h_f, r_f), \tag{7}$$

under $G \supset H \times R$. The representations for the left-handed (right-handed) fermions are h_fs whose corresponding r_fs are found in $\sigma_{1i}(\sigma_{2i})$ obtained in Eq. (6).

A phenomenologically acceptable model needs chiral fermions in

four dimensions as the SM does. The SO(1, 7) spinor is not self-dual and its charge conjugate state is in a different representation from itself. Thus the Majorana condition cannot be used to obtain a chiral structure from a vectorlike representation of G. Therefore, we need to introduce complex representation for eight-dimensional fermions. Thus eight-dimensional model possesses a completely different feature from $4n + 2$-dimensional models. We must work on complex representation for eight-dimensional fermions.

Finally coset space S/R of our interest should satisfy rank S = rank R to generate chiral fermions in four dimensions [43]. We list all of four-dimensional coset spaces S/R satisfying the condition and decompositions of SO(4) spinor and vector representation in Table I.

III. SEARCH FOR CANDIDATES

In this section we search for realistic models in the CSDR scheme in eight-dimensions.

First we investigate four-dimensional gauge group H and higher-dimensional gauge group G for each coset space of (i), (ii) and (iii) listed in Table. I. The four-dimensional gauge groups acceptable for H are listed in Table. II. This Table is obtained from the following considerations.

1. The number of U(1)s in H must be more than that in R, since the U(1)s in R are also part of its centralizer, i.e. part of H. Therefore, the number of U(1)s in H must be R or more. We thus exclude SU(5), SO(10), and E$_6$ for coset space of (ii) and G_{SM}, SU(5), SO(10), E$_6$, SU(5) × U(1), SO(10) × U(1), and E$_6$ × U(1) for coset space of (iii).

2. We also exclude G_{SM} for the coset space of (ii) and $G_{\mathrm{SM}} \times$ U(1) for the coset space of (iii). The hypercharges of the SM should be reproduced by the U(1) charges in R, which means that all the hypercharges must appear in the decomposition of SO(4) spinor. The dimension of the SO(4) spinor representation is however two, and hence more than two different values of U(1) charges are not available. Consequently, these cases never reproduce the five hypercharges of the SM fermions.

3. We allow at most one extra U(1) in four-dimensional gauge group. This excludes $G_{\mathrm{SM}} \times$ U(1), SU(5) × U(1), SO(10) × U(1), E$_6$ × U(1), $G_{\mathrm{SM}} \times$ U(1) × U(1), SU(5) × U(1) × U(1), SO(10) × U(1) × U(1), and E$_6$ × U(1) × U(1) for coset space of (i), and $G_{\mathrm{SM}} \times$ U(1) × U(1), SU(5) × U(1) × U(1), SO(10) × U(1) × U(1), and E$_6$ × U(1) × U(1) for coset space of (ii).

The higher-dimensional gauge group G should have the same rank as that of $H \times R$ up to U(1)s and possess complex representations to obtain chiral fermions. We also list the candidates of G in Table II.

TABLE II: The candidates of H and G.

S/R	H	G
(i)	SU(3) × SU(2) × U(1)	SU(7), E_6
	SU(5)	SU(7), E_6
	SO(10)	SU(8), SO(14)
	E_6	SU(9)
(ii)	SU(3) × SU(2) × U(1) × U(1)	SU(7), E_6
	SU(5) × U(1)	SU(7), E_6
	SO(10) × U(1)	SU(8), SO(14)
	E_6 × U(1)	SU(9)
(iii)	SU(3) × SU(2) × U(1) × U(1) × U(1)	SU(7), E_6
	SU(5) × U(1) × U	SU(7), E_6
	SO(10) × U(1) × U	SU(8), SO(14)
	E_6 × U(1) × U	SU(9)

We investigate representations of four-dimensional gauge group in the CSDR scheme. The representations for scalars in four-dimensional spacetime are obtained by comparing Eq. (3) and Eq. (4), while those for fermions are similarly obtained by comparing Eq. (6) and Eq. (7). Note again that F must be complex representation in order to obtain chiral fermions in four dimensions. We limit the dimension of F to 1000 to avoid numerous representations of the fermions under four-dimensional gauge group.

Exhaustive investigation of all combinations of S/R, G, H and F leaves six candidates of models which include at least one generation of known fermions; they are listed in Table III.

TABLE III: Four-dimensional scalar and fermion representations for each combination of S/R, G, H and F.

S/R	H	G	scalar	F	fermions
(i)	SO(10)	SO(14)	$\mathbf{10}$	$\mathbf{64}$	$\{\mathbf{16}\}^2$
				$\mathbf{832}$	$\{\mathbf{16}\}^2$, $\{\overline{\mathbf{144}}\}^2$
(ii)	SO(10) × U(1)	SO(14)	$\mathbf{10}(1)$, $\mathbf{10}(-1)$	$\mathbf{64}$	$\mathbf{16}(0)$, $\mathbf{16}(1)$, $\mathbf{16}(-1)$
				$\mathbf{832}$	$\{\mathbf{16}(0)\}^2$, $\mathbf{16}(1)$, $\mathbf{16}(-1)$, $\overline{\mathbf{144}}(0)$, $\overline{\mathbf{144}}(1)$, $\overline{\mathbf{144}}(-1)$
(iii)	SO(10) × U(1) × U(1)	SO(14)	$\mathbf{10}(1,1)$, $\mathbf{10}(1,-1)$ $\mathbf{10}(-1,1)$, $\mathbf{10}(-1,-1)$	$\mathbf{64}$	$\mathbf{16}(1,0)$, $\mathbf{16}(-1,0)$, $\mathbf{16}(0,1)$, $\mathbf{16}(0,-1)$
				$\mathbf{832}$	$\{\mathbf{16}(1,0)\}^2$, $\{\mathbf{16}(-1,0)\}^2$, $\{\mathbf{16}(0,1)\}^2$, $\{\mathbf{16}(0,-1)\}^2$, $\overline{\mathbf{144}}(1,0)$, $\overline{\mathbf{144}}(-1,0)$, $\overline{\mathbf{144}}(0,1)$, $\overline{\mathbf{144}}(0,-1)$

For coset space of (i), we embed $R = \mathrm{SU}(2) \times \mathrm{SU}(2)$ into $G = \mathrm{SO}(14)$ according to the decomposition

$$\mathrm{SO}(14) \supset \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{SO}(10). \tag{8}$$

The decomposition of the adjoint representation of SO(14) according to the decomposition of Eq. (8) is

$$\mathbf{91} = (\mathbf{1},\mathbf{1},\mathbf{45}) + (\mathbf{3},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{3},\mathbf{1}) + (\underline{\mathbf{2},\mathbf{2}},\mathbf{10}), \tag{9}$$

and thus we obtain $\mathbf{10}$ as the scalar reperesentation in four dimensions. Similarly, we decompose the complex representations $\mathbf{64}$ and $\mathbf{832}$ of SO(14) according to the decomposition of Eq. (8) as

$$\mathbf{64} = (\underline{\mathbf{2},\mathbf{1}},\mathbf{16}) + (\underline{\mathbf{1},\mathbf{2}},\overline{\mathbf{16}}), \tag{10}$$

$$\begin{aligned}\mathbf{832} =& (\underline{\mathbf{1},\mathbf{2}},\mathbf{144}) + (\underline{\mathbf{2},\mathbf{1}},\overline{\mathbf{144}}) + (\mathbf{2},\mathbf{3},\mathbf{16}) + (\mathbf{3},\mathbf{2},\overline{\mathbf{16}}) \\ &+ (\underline{\mathbf{1},\mathbf{2}},\overline{\mathbf{16}}) + (\underline{\mathbf{2},\mathbf{1}},\mathbf{16}),\end{aligned} \tag{11}$$

and obtain $\{\mathbf{16}\}^2$ from $F=\mathbf{64}$ and $\{\mathbf{16}\}^2+\{\overline{\mathbf{144}}\}^2$ from $F=\mathbf{832}$ as representations for the left-handed fermion in four dimensions.

For coset space of (ii), we embed SU(2) × U(1) into SO(14) according to the decomposition

$$\begin{aligned}\mathrm{SO}(14) &\supset \mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{SO}(10)\\ &\supset \mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{SO}(10).\end{aligned} \tag{12}$$

The decomposition of the adjoint representation of SO(14) according to the decomposition of Eq. (12) is

$$\begin{aligned}\mathbf{91} =&(\mathbf{1},\mathbf{1},\mathbf{45})+(\mathbf{3},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{3},\mathbf{1})+(\mathbf{2},\mathbf{2},\mathbf{10})\\ =&(\mathbf{1},\mathbf{45})(0)+(\mathbf{3},\mathbf{1})(0)+(\mathbf{1},\mathbf{1})(2)+(\mathbf{1},\mathbf{1})(0)+(\mathbf{1},\mathbf{1})(-2)\\ &+(\underline{\mathbf{2}},\mathbf{10})(\underline{1})+(\underline{\mathbf{2}},\mathbf{10})(\underline{-1}),\end{aligned} \tag{13}$$

and thus we obtain $(\mathbf{10}(1))$ and $(\mathbf{10}(-1))$ as the scalar representations in four dimensions. Similarly, we decompose the complex representations $\mathbf{64}$ and $\mathbf{832}$ of SO(14) according to the decomposition of Eq. 12) as

$$\begin{aligned}\mathbf{64} =&(\mathbf{2},\mathbf{1},\mathbf{16})+(\mathbf{1},\mathbf{2},\overline{\mathbf{16}})\\ =&(\mathbf{2},\mathbf{16})(0)+(\mathbf{1},\overline{\mathbf{16}})(1)+(\mathbf{1},\overline{\mathbf{16}})(-1)\end{aligned} \tag{14}$$

$$\begin{aligned}\mathbf{832} =&(\mathbf{1},\mathbf{2},\mathbf{144})+(\mathbf{2},\mathbf{1},\overline{\mathbf{144}})+(\mathbf{2},\mathbf{3},\mathbf{16})+(\mathbf{3},\mathbf{2},\overline{\mathbf{16}})\\ &+(\mathbf{1},\mathbf{2},\overline{\mathbf{16}})+(\mathbf{2},\mathbf{1},\mathbf{16})\\ =&(\underline{\mathbf{1}},\mathbf{144})(\underline{1})+(\underline{\mathbf{1}},\mathbf{144})(\underline{-1})+(\underline{\mathbf{2}},\overline{\mathbf{144}})(\underline{0})+(\mathbf{2},\mathbf{16})(2)\\ &+(\underline{\mathbf{2}},\mathbf{16})(\underline{0})+(\mathbf{2},\mathbf{16})(-2)+(\mathbf{3},\overline{\mathbf{16}})(1)+(\mathbf{3},\overline{\mathbf{16}})(-1)\\ &+(\underline{\mathbf{1}},\overline{\mathbf{16}})(\underline{1})+(\underline{\mathbf{1}},\overline{\mathbf{16}})(\underline{-1})+(\underline{\mathbf{2}},\mathbf{16})(\underline{0}),\end{aligned} \tag{15}$$

and obtain $\mathbf{16}(0)$, $\mathbf{16}(1)$ and $\mathbf{16}(-1)$ from $F=\mathbf{64}$ and $\{\mathbf{16}(0)\}^2$, $\mathbf{16}(1)$, $\mathbf{16}(-1)$, $\overline{\mathbf{144}}(0)$, $\overline{\mathbf{144}}(1)$ and $\overline{\mathbf{144}}(-1)$ from $F=\mathbf{832}$ as representations for the left-handed fermion in four dimensions.

For coset space of (iii), we embed U(1) × U(1) into SO(14) according to the decomposition

$$\begin{aligned}\mathrm{SO}(14) &\supset \mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{SO}(10)\\ &\supset \mathrm{SO}(10)\times\mathrm{U}(1)\times\mathrm{U}(1).\end{aligned} \tag{16}$$

The decomposition of the adjoint representation of SO(14) according to the decomposition of Eq. (16) is

$$\begin{aligned}\mathbf{91} =&(\mathbf{1},\mathbf{1},\mathbf{45})+(\mathbf{3},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{3},\mathbf{1})+(\mathbf{2},\mathbf{2},\mathbf{10})\\ =&\mathbf{45}(0,0)+\mathbf{1}(2,0)+\mathbf{1}(0,0)+\mathbf{1}(-2,0)+\mathbf{1}(0,2)+\mathbf{1}(0,0)\\ &+\mathbf{1}(0,-2)+\mathbf{10}(1,1)+\mathbf{10}(1,-1)+\mathbf{10}(-1,1)+\mathbf{10}(-1,-1),\end{aligned} \tag{17}$$

and thus we obtain $(\mathbf{10}(1,1))$, $(\mathbf{10}(1,-1))$, $(\mathbf{10}(-1,1))$ and $(\mathbf{10}(-1,-1))$ as the scalar representations in four dimensions. Similarly, we decompose the complex representations $\mathbf{64}$ and $\mathbf{832}$ of SO(14) according to the decomposition of Eq. (16) as

$$\begin{aligned}\mathbf{64} =&(\mathbf{2},\mathbf{1},\mathbf{16})+(\mathbf{1},\mathbf{2},\overline{\mathbf{16}})\\ =&\mathbf{16}(1,0)+\mathbf{16}(-1,0)+\overline{\mathbf{16}}(0,1)+\overline{\mathbf{16}}(0,-1)\end{aligned} \tag{18}$$

$$\begin{aligned}\mathbf{832} =&(\mathbf{1},\mathbf{2},\mathbf{144})+(\mathbf{2},\mathbf{1},\overline{\mathbf{144}})+(\mathbf{2},\mathbf{3},\mathbf{16})+(\mathbf{3},\mathbf{2},\overline{\mathbf{16}})+(\mathbf{1},\mathbf{2},\overline{\mathbf{16}})\\ &+(\mathbf{2},\mathbf{1},\mathbf{16})\\ =&\mathbf{144}(0,1)+\mathbf{144}(0,-1)+\overline{\mathbf{144}}(1,0)+\overline{\mathbf{144}}(-1,0)\\ &+\mathbf{16}(1,2)+\mathbf{16}(1,0)+\mathbf{16}(1,-2)+\mathbf{16}(-1,2)\\ &+\mathbf{16}(-1,0)+\mathbf{16}(-1,-2)+\overline{\mathbf{16}}(2,1)+\overline{\mathbf{16}}(2,-1)\\ &+\overline{\mathbf{16}}(0,1)+\overline{\mathbf{16}}(0,-1)+\overline{\mathbf{16}}(-2,1)+\overline{\mathbf{16}}(-2,-1)\\ &+\overline{\mathbf{16}}(0,1)+\overline{\mathbf{16}}(0,-1)+\mathbf{16}(1,0)+\mathbf{16}(-1,0),\end{aligned} \tag{19}$$

and obtain $\mathbf{16}(1,0)$, $\mathbf{16}(-1,0)$, $\mathbf{16}(0,1)$ and $\mathbf{16}(0,-1)$ from $F=\mathbf{64}$ and $\{\mathbf{16}(1,0)\}^2$, $\{\mathbf{16}(-1,0)\}^2$, $\{\mathbf{16}(0,1)\}^2$, $\{\mathbf{16}(0,-1)\}^2$, $\overline{\mathbf{144}}(1,0)$, $\overline{\mathbf{144}}(-1,0)$, $\overline{\mathbf{144}}(0,1)$ and $\overline{\mathbf{144}}(0,-1)$ from $F=\mathbf{832}$ as representations for the left-handed fermion in four dimensions.

We can obtain one generation of the SM fermion from all of the candidates listed in Table III since the representations **16** and **144** of SO(10) include one generation of the SM fermion. The models with SU(3)/SU(2)×U(1) are particularly intersting in our results. We obtain the three generations of the SM fermions for this coset space with $F = \mathbf{64}$. We also obtain odd number generation in the combination of coset space of (ii) with $F = \mathbf{832}$. This is due to the fact that the SO(4) spinor is not self conjugate which forbid Majorana-Weyl condition and that R, which is SU(2)×U(1), are embedded into SO(4) lopsidedly.

IV. SUMMARY AND DISCUSSIONS

We analyzed the gauge-Higgs unification models based on eight-dimensional gauge theories under the coset space dimensional reduction and exhaustively searched for models which lead to a phenomenologically promising model after dimensional reduction.

We first made a complete list of the eight-dimensional models by determining the structure of the coset space S/R, the gauge group G, and the representations F of G for fermions. We obtained a full list of the possible coset space S/R in Table I by requiring dimS/R =4 and rank$S$=rank$R$. The gauge group G is required to have complex representations and to lead to one of the following two-types of gauge groups after dimensional reduction: the GUT-like gauge groups such as E_6, SO(10), SU(5) and these groups with one or two extra U(1)s, or the Standard-Model (SM)-like groups which are G_{SM} =SU(3)×SU(2)×U(1) and G_{SM} with one or two extra U(1)s (see table. II). The representation F for fermions must be a complex representation of G and is limited to be dimension less than 1000.

We then analyzed the particle contents of the four-dimensional theories that are obtained from each of the candidates (S/R,G,F). We found the phenomenologically promising models which induce H=SO(10)(× one or two U(1)s) in four-dimensions, while other cases are found to be unsuccessful such as H=SU(3)×SU(2)×U(1), H=SU(5), and H=E_6(with one or two extra U(1)s) in four-dimensions. We summarized these models in Table III. The SO(10) GUT-like models provide more than two fermions of **16** and **144** representations, along with a number of scalars of **10** representation. A scalar of **10** can be interpreted as the electroweak Higgs particles. More than two fermions of **16** and **144** representations would provide the generations of the fermions in the SM. The most interesting model in this regard is the one with S/R = SU(3)/SU(2)×U(1), G=SO(14), F=**64** and H=SO(10)×U(1). This model leads three fermions of **16**, suggesting the existence of three generations of the SM fermions.

An apparent challenge is to break the SO(10) gauge symmetry down to the SM ones. This difficulty can be overcome by employing the Hosotani mechanism, also known as the Wilson flux breaking mechanism [45, 46, 47], or non-trivial boundary conditions of S/R [48]. We leave further analysis for future study.

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